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VELOCITIES OF ELASTIC WAVES IN CONSOLIDATED GRANULAR MEDIA

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Zaikin [1] proposed a method of calculating the effective moduli of dry consolidated granular media on the basis of the solution of the problem of the elastic deformation of an individual grain. The model of intersecting spheres (MIS) was used to describe the structure of the pore space. A numerical procedure for calculating the velocities of elastic waves was also presented. As a result, a functional relationship was established between the velocities of longitudinal and transverse waves and the parameters of the pore space: porosity f and the product of the specific surface (per unit volume) and mean grain size $\eta = S_V\langle D\rangle$. Equations were presented for the velocities of the elastic waves in the case when the Poisson ratio of the material of the grains $\sigma = 0.25$.

In the present study, we use the approach referred to above to study the effect of the elastic properties of the solid phase on the effective elastic moduli of a granular medium. Results are presented from experimental studies of the structure of the pore space and the velocities of ultrasonic waves. These studies were conducted using specially prepared three-dimensional models of granular media.

As was established previously, the dependence of the effective moduli on the elastic parameters of the solid phase is determined solely by the Poisson's ratio of the latter. In our calculations, it varied within the range $0.05 \le \sigma \le 0.45$, with increments of 0.05. For each value of σ, we obtained the dependence of the velocities of the elastic waves on porosity and the type of packing of the grains (the number of contacts). Similar relations for $\sigma = 0.344$ are shown below. We used multiple regression analysis to approximate these relations with equations of the form

$$V_P^*/V_P = 1 - A_1 f - A_2 \eta, \quad V_S^*/V_S = 1 - A_3 f - A_4 \eta. \tag{1}$$

The asterisks denote effective elastic parameters of the granular medium. We used nine values of σ to construct the regression equations for the coefficients of Eq. (1). Approximation of the coefficients A_1, A_2, A_3, A_4 by a cubic parabola gives results which are quite satisfactory (mean error no greater than 0.7%):

$$\begin{aligned} A_1 &= 0{,}722 + 0{,}029\sigma + 0{,}303\sigma^2 - 0{,}268\sigma^3, \\ A_2 &= 0{,}058 + 0{,}123\sigma - 0{,}494\sigma^2 + 1{,}16\sigma^3, \\ A_3 &= 0{,}685 + 0{,}0201\sigma + 0{,}05\sigma^2 + 0{,}147\sigma^3, \\ A_4 &= 0{,}0664 - 0{,}0225\sigma + 0{,}0277\sigma^2 - 0{,}133\sigma^3. \end{aligned} \tag{2}$$

With the use of Eqs. (1) and (2) to represent numerical results on the velocities of the elastic waves, the mean error of the prediction is no greater than 8%.

Before analyzing the expressions that have been obtained, let us discuss the MIS. The geometry of this model is determined by two dimensionless parameters. Since we are using f

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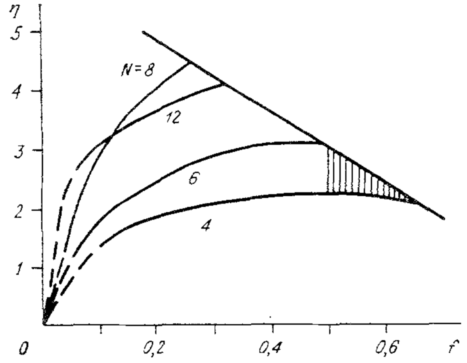

Fig. 1

and η in our investigation, we will find the region of existence of the MIS in these coordinates. The parameters f and S_V were represented analytically in [1] through the number and radius of the contacts between grains, but this is valid only if the contacts do not intersect one another. This case is shown by the solid lines in Fig. 1. The numbers next to the curves correspond to the type of packing (the number of contacts N). Fairly complicated geometric constructions must be completed in order to continue the function η(f, N) into the region of intersecting contacts. It is clear that an increase in the radius of the contacts will ultimately transform the granular medium into a continuum with zero porosity and a zero specific surface, i.e., η(0, N) = 0. In the intermediate region between this point and the region corresponding to the analytical representation, the function η can be reconstructed by interpolation. The dashed lines in Fig. (1) show the result obtained using a cubic spline. Here, only the terms of the spline function which are linear with respect to f are significant in the neighborhood of the coordinate origin. The maximum value of the derivative is seen for the packing with 12 contacts η'(0, 12) = 61.5. On the other hand, the curve η(f, 4) satisfies the condition $\eta \geq 3.09f$. In the case of point contact between spheres for any type of packing, the relation η = 6(1 − f) (a straight line). It can be seen from Fig. 1 that the region of existence of the MIS lies within a triangle with the sides:

$$\eta = 61{,}5f,\ \eta = 3{,}09f,\ \eta = 6(1 - f). \tag{3}$$

For all values of f and η lying within triangle (3) and, thus, for the entire region of existence of the MIS, an increase in the Poisson's ratio of the solid phase leads to an increase in V_S^*/V_S and a decrease in V_P^*/V_P. Here, for the parameter $\gamma^* = V_S^*/V_P^*$ – widely used in seismic prospecting – the relation $\gamma^* > \gamma$ is valid at all values of f and η satisfying the MIS and at $\sigma > 0.067$. If $0 \leq \sigma \leq 0.067$, then $\gamma^* \leq \gamma$ in that part of the MIS region above the straight line η = bf ($b = (A_1 - A_3)/(A_4 - A_2)$, with b changing from 4.4 to 61.5).

An increase in γ^* has an interesting result. It is known that $\sigma = 0.5(1 - 2\gamma^2)/(1 - \gamma^2)$. At $\gamma > 1/\sqrt{2}$, the Poisson's ratio is negative. The thermodynamics of the linear theory of elasticity does not require σ to be positive, but there are presently no known solids with $\sigma < 0$. Nevertheless, negative values of σ^* were obtained in experiments conducted in [2] on highly porous gas-saturated sandstones.

It follows from Eqs. (1) and (2) that the effective Poisson's ratio of a granular medium can be negative. In the hatched region of Fig. 1, $\gamma^* > 0.707$. Here, it was assumed that σ = 0.05. The existence of a region of anomalous values of γ^* is possible only at $\sigma < 0.16$. In the coordinates f and η, the region of negative σ^* has the form of a curvilinear triangle with sides formed by the curves η = 6(1 − f) and η(f, 4). The position of the third side is determined by the Poisson's ratio of the solid phase. Here, f exceeds 0.47 within this region in every case.

There have recently appeared studies [3, 4] in which it was concluded that in the case of a very significant gradient of elastic properties of the matrix and inclusions (for example, in a solid–gas), the averaged equations of motion of the heterogeneous medium do not coincide with the equations of motion of the matrix material. In our view, the existence of negative Poisson's ratios for granular media (as opposed to linearly elastic media) is the result of the description of the former – which have equations of motion that differ from the linearly elastic equations in the context of the linear theory of elasticity.

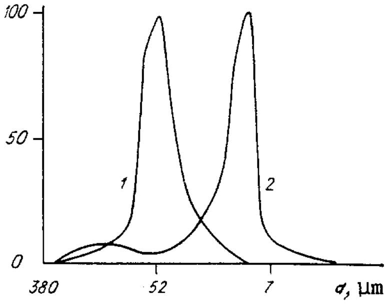

Fig. 2

We used the method of sintering to prepare three-dimensional models of granulated polymethyl methacrylate (PMMA) in order to empirically study the propagation of elastic waves in granular media. The dimensions of the models were 30 × 20 × 10 cm. The PMMA granules were nearly ideal spheres, with a mean size of 162.7 μm. The standard deviation of the granule-sized distribution was 1.67 μm. The velocities of the elastic waves in the PMMA: V_P = 2730 m/sec; V_S = 1333 m/sec. The density $\rho = 1.28 \cdot 10^6$ g/m³.

By changing the sintering regime (temperature, time, and pressure), we obtained models having pore spaces with different structure parameters. Porosity and specific surface were determined from curves of capillary pressure (within the framework of the model of cylindrical pores in [5]). Here, we used an Autopore-9200 mercury porometer. For the group of 13 models, the range of f was 0.28-0.39. The specific surface per unit of mass $S_m = S_V/\rho(1 - f)$ ranged from 0.0195 to 0.0382 m²/g.

Figure 2 shows the pore-size distribution (pore diameter d is plotted off the x axis in a logarithmic scale, while the numbers plotted off the y axis represent the percentage of pores of the given diameter in the total volume of the pore space) in the model with f = 0.36 and S_m = 0.0281 m²/g (curve 1). This type of distribution was characteristic of all of the models. Shown for comparison is the pore-size distribution in a specimen of sandstone from the Romashkinsk deposit in the Tatar Republic. This sandstone has a low content of clay minerals, with f = 0.22 and S_m = 0.0238 m²/g (curve 2). It is not hard to see that the models composed of granulated PMMA satisfactorily approximate the features of the pore-space structure of actual sandstones.

The mean pore diameter for the group of models ranges from 46 to 76 μm. For globular systems, Zagrafskaya et al. [6] proposed the following semi-empirical relation to link particle size, pore diameter, and the mean number of contacts per particle:

$$N = 1{,}63\ \langle D \rangle / d + 2{,}62. \tag{4}$$

In our case, N changes from 6.1 to 8.34.

The velocities of the longitudinal and transverse waves were measured by a digital system consisting of blocks to excite and record ultrasonic vibrations. The operation of the blocks was controlled by an SM-4 computer. The source was secured to the end of the model, while the detector moved over the top surface. The measurements were made by the profile method. The increments in the profile were 1 cm, with 10-15 increments. The velocity measurement error was no greater than 0.5%. Most of the pulse frequencies were within the range 40-80 kHz. A three-dimensional situation was realized with such frequencies and model dimensions. In the least favorable situation, the ratio of wavelength to grain size $\lambda/\langle D \rangle \geq 40$.

Figures 3 and 4 show the results of the velocity measurements on the models. The values of velocity are referred to the velocities in the PMMA. With allowance for the number of contacts obtained from (4), it can be concluded that the theoretical data agrees with the experimental results. In the determination of pore-space structure from the MIS, the dimensionless variables f, η, N (as any other variables) are equivalent and each is expressed in terms of the other two variables. In order to be able to compare the results of the measurements with (1), we used the known values of S_m, f, and ⟨D⟩ to calculate η for each model. Then using multiple regression analysis, we can represent the velocities of the elastic waves in the form of linear equations:

$$V_P^*/V_P = 1 - 1{,}6f - 0{,}025\ \eta, \quad V_S^*/V_S = 1 - 1{,}6f - 0{,}0098\ \eta,$$

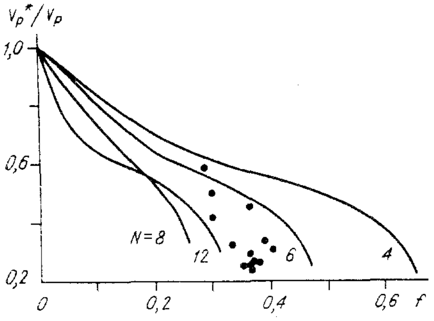

Fig. 3

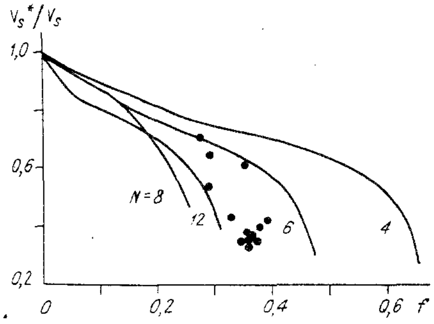

Fig. 4

with correlation coefficients of 0.95 and 0.89, respectively, and a mean error of 10%. The velocity of the longitudinal waves is more sensitive to the structure of the pore space. This results in an increase of the parameter γ: whereas $\gamma = 0.488$ for PMMA, for the models γ^*/γ changes from 1.16 to 1.32.

For comparison with the theoretical values, we take the following coefficient values from (2): $A_1 = 0.76$, $A_2 = 0.089$, $A_3 = 0.7$, $A_4 = 0.057$. Although the differences are quite large, it can be seen that the structure of the pore space has a greater effect on the velocity of longitudinal waves. Leaving aside the assumptions made for the theoretical constructions, it can be suggested that the quantitative differences between the coefficients are due to the fact that a cylindrical-pore model was used to determine specific surface.

Thus, we have obtained equations linking the velocities of elastic waves in dry consolidated granular media with the Poisson's ratio of the solid phase and structural parameters of the pore space – porosity and the product of specific surface and mean grain size. It was shown that such media can have negative Poisson's ratios. We used models of a granular medium composed of granulated PMMA to experimentally construct functions connecting the velocities of elastic waves with parameters of the pore-space structure. These functions qualitatively agree with the corresponding theoretical results.

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